No Information Is Lost: a Revisit of Black Hole Information Loss Paradox

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We revisit in detail the paradox of black hole information loss due to Hawking radiation as tunneling. We compute the amount of information encoded in correlations among Hawking radiations for a variety of black holes, including the Schwarzchild black hole, the Reissner-Nordström black hole, the Kerr black hole, and the Kerr-Newman black hole. The special cases of tunneling through a quantum horizon and geometrically non-commutative black holes are also considered. Within a phenomenological treatment based on the accepted emission probability spectrum from a black hole, we find that information is leaked out hidden in the correlations of Hawking radiation. The recovery of this previously unaccounted for information helps to conserve the total entropy of a system composed of a black hole plus its radiations. We thus conclude, irrespective of the microscopic picture for black hole collapsing, the associated radiation process: Hawking radiation as tunneling, must be a unitary process.

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INTRODUCTION

Since Hawking radiation was first discovered [1, 2], its inconsistency with quantum theory has been widely debated. Within the scenario developed along the original work of Hawking [2, 3], irrespective of the initial state of the composing matter, it always evolves into a thermal state after collapsing into a black hole. Such a picture leads to a paradoxical claim of black hole information loss or the violation of entropy conservation, as different initial states: pure or mixed, evolve into the same final thermal state. This directly violates the principle of unitarity for quantum dynamics of an isolated system and brings a serious challenge to the foundations of modern physics. The lack of a resolution on this fundamental problem concerning thermodynamics, relativity, quantum mechanics, and cosmology, has attracted considerable attention [4]. The more recent memory includes the famous betting of several passionate physicists [5]. This problem is now generally known as "the paradox of black hole information loss", and is attributed to the semiclassical nature of the treatment for Hawking radiation by some [4]. In the past few decades, several methods [3, 6, 7, 8, 9, 10] have been suggested for resolving this paradox; none has been successful. In fact, each failed attempt for a resolution seems to have made the existence of this paradox more serious and attracted more interest, especially after the possibility that information about infallen matter may hide inside the correlations between the Hawking radiation and the internal states of a black hole was ruled out. It seems either unitarity or Hawking's semiclassical treatment of radiation must break down [11].

In most previous resolutions to the black hole information loss paradox [3, 6, 7, 8, 9, 10, 11], the radiation from a black hole is considered purely thermal because the background geometry is fixed and energy conservation is not enforced during the radiation process. On the other hand, some have correctly argued that information could come out if the emitted radiations were not exactly thermal but instead the radiation spectrum contains a subtle non-thermal correction [12]. The semiclassical derivation of Hawking is based on Bogoliubov transformation which always gives pure thermal radiations. Although Hawking later explained the particles of radiation as stemming from vacuum fluctuations tunneling through the horizon of a black hole together with Hartle [13], the original semiclassical treatment did not have any direct connection with tunneling. It is thus important to review the picture of Hawking radiation as tunneling, whereby a pair of particles is spontaneously generated inside the horizon. The positive energy particle tunnels out to the infinity while the negative energy one remains in the black hole. Alternatively, the positive and negative energy pair is created outside the horizon, and the negative energy particle tunnels into the black hole because its orbit exists only inside the horizon, while the positive energy one remains outside and emerges at infinity.

Recently, Parikh and Wilczek [14] developed the method of Hawking radiation as tunneling due to Kraus and Wilczek [15, 16]. The flux of particles can be computed directly within their tunneling picture. Extensions to many other situations have since been made [17, 18, 19, 20, 21, 22], that established a firm basis for the physical explanation of Hawking radiation as tunneling. In their current version [14, 23, 24], energy conservation is directly enforced and is observed to play an important role. The actual calculation benefits from a coordinate system, the Painlevé coordinate,

which unlike the Schwarzchild coordinate, is regular at the horizon, and is considered to be a particularly suitable and convenient choice. The tunneling process involves particles that can be supplied by considering the geometrical limit because of the infinite blueshift of the outgoing wave-packets near the horizon. The barrier is created by the outgoing particle itself, which is ensured by energy conservation. Finally, the radial null geodesic motion is considered, and the WKB approximation is used to obtain a tunneling probability $\Gamma \sim \exp\left[-2\mathrm{Im}(I)\right]$ related to the imaginary part of the action I of the black hole. This tunneling probability is related to the change of the entropy of a black hole [14]. It is clearly non-thermal. As was pointed out by us in an earlier communication [25], this non-thermal feature implies the existence of information-carrying correlations among sequentially emitted Hawking radiations.

In a recent communication [25], we visited the topic of the black hole information loss paradox starting with the non-thermal spectrum obtained by Parikh and Wilczek [14]. For a typical black hole, a Schwarzchild black hole, we discover the existence of correlation among Hawking radiations. Upon carefully evaluating the amount of information carried away in this correlation, we find there is simply no loss of information and that the entropy is conserved for the total system of the black hole plus its radiations. The information coded into the correlations and carried away by the Hawking radiation is found to balance exactly the amount claimed lost previously [25]. Our result thus shows that tunneling through the horizon is an entropy-conserving process when the entropy associated with the correlations of emitted particles is included. This conclusion is consistent with the principle of unitarity for quantum mechanics concerning an isolated system.

The purpose of this article is to revisit the paradox of black hole information loss, along the ideas originally developed by us in the earlier communication [25]. In addition to provide more details for an indepth understanding and to support our original claim, we also extend our previous analysis from the Schwarzschild black holes to the Reissner-Nordström black holes, the Kerr black holes, and the Kerr-Newman black holes. For charged black holes, both tunnelings due to massless neutral particles and massive charged particles will be considered. We find: (a) sequentially tunneled particles are correlated with each other, and (b) the entropy of the total system composed of a black hole and its radiations is conserved provided correlations between Hawking radiations are included. For massless neutral particles, due to charge conservation the black hole will evolve into the extreme case where the temperature is zero and the radiation vanishes. The total entropy remains conserved during the evolution. Before the extreme case arrives, all our analysis remain appropriate and the correlation is found to be capable of taking the information out from a charged black hole as we suggested. In the extreme case, the total information remains conserved, although some may have leaked out while others stay in the stable extreme black hole. In order to avoid the extreme case we analyze the tunneling of massive charged particles from the Reissner-Nordström black holes and we verify that the entropy conservation still holds due to energy and charge conservations. For rotating black holes and charged rotating black holes, the same conclusion is reached: information is carried out by correlations among the non-thermal radiations; the total entropy is conserved and no information is lost. Thus the Hawking radiation as tunneling in the above cases must also be unitary. Additionally, we discuss Hawking radiation as tunneling through a quantum horizon where a black hole may evolve into a remnant and not evaporate at all. This case does not conflict with our claim of no information loss either, as information is still carried away by the correlations between outgoing particles. When geometrical non-commutativity is incorporated into the Schwarzschild spacetime and with the tunneling probability obtained in the background, our discussion is found to remain applicable. Total information remains conserved for the non-thermal radiation in a non-commutative space.

Summarizing our work reported in this paper, for an extensive list of black holes, we find the inclusion of the correlations hidden inside non-thermal Hawking radiations resolves the paradox of black hole information loss. We find the total entropy is always conserved, significantly extending our earlier claim [25]. We thus have provided a self-consistent and reasonable resolution to the paradox of black hole information loss. At the heart of our resolution is the existence of correlations among emissions of Hawking radiation. Independent of the microscopic picture of how a black hole actually collapses and what its initial and final states are, a non-thermal emission spectrum implies the existence of correlation. As we show below in detail, for various types of black holes, after carefully counting the total correlation we find the total entropy is conserved and no information loss occurs. This puts our claim that Hawking radiation must be a unitary process on a firm ground.

Our paper is organized as follows. In section II we first review our method of resolving the paradox for a Schwarzschild black hole in the picture of Hawking radiation as tunneling [25]. Subsequently, this method is applied to the analysis of radiation as tunneling for Reissner-Nordström black holes, Kerr black holes, and Kerr-Newman black holes. In the third section, we discuss information loss paradox for Hawking radiation as tunneling through a quantum horizon. The case of non-commutative black holes is discussed in section IV. Section V ends with conclusions and some remarks. To simplify the expressions and calculations, we take the convenient units of $k = \hbar = c = G = 1$.

HAWKING RADIATION AS TUNNELING THROUGH A CLASSICAL HORIZON

We start with a brief review of the method and ideas we introduced earlier for resolving the paradox of black hole information loss [25]. As was pointed out before, we find that correlations exist among Hawking radiations from a Schwarzchild black hole if its radiation spectrum is non-thermal as required by energy conservation. A careful counting of the total correlations is shown by us to balance exactly the information previously considered lost. After the review, we will perform analogous analysis for other situations such as Reissner-Nordström black holes, Kerr black holes, and Kerr-Newman black holes.

Schwarzchild black hole

To properly describe any phenomena involves the crossing of horizon, it is helpful to change from the Schwarzchild coordinates into Painlevé coordinates, which are not singular at the horizon. Finding the radial null geodesic and computing the imaginary part of the action for the process of s-wave emission across the horizon, the tunneling probability is found to be [14]

$$\Gamma \sim \exp\left[-8\pi E\left(M-\frac{E}{2}\right)\right] = \exp\left(\Delta S\right), \tag{1}$$

where the second equal sign expresses this result in terms of the change of the Bekenstein-Hawking entropy [1, 2, 27] for the Schwarzchild black hole $S_{\rm BH}=A/4=4\pi M^2$, where $A=4\pi(2M)^2$ is the surface area of a Schwarzchild black hole with mass M and radius $2M$. It is important to note that this spectrum is non-thermal, different from the thermal case of a simple exponential $\Gamma(E)=\exp\left(-8\pi EM\right)$. A non-thermal spectrum implies that individual emissions are correlated because the emission probability for two simultaneous emissions is not the same as the product probabilities for two independent emissions. This point has been outlined in detail in an earlier communication [25], where we show that such correlations can encode information, thus leading to information being carried away by Hawking radiations. When the amount of information carried away by correlation is included, the total entropy of the system composed of the black hole and Hawking radiation is conserved. In other words the non-thermal spectrum suggests unitarity and no information loss in black holes.

In statistical theory [26], if the probability for two events arising simultaneously is identically equal to the product probability of each event arising independently, these two events are independent and there exists no correlation between them. Otherwise, they are correlated. Accepting the spectrum of Eq. (1), the probability for the first emission at an energy E_1 becomes

$$\Gamma(E_1) \;=\; \exp\left[-8\pi E_1\left(M-\frac{E_1}{2}\right)\right]. \tag{2}$$

After this first emission, the mass of the black hole is reduced to $M-E_1$ due to energy conservation. The conditional probability for a second emission at an energy E_2 is therefore given by

$$\Gamma(E_2|E_1) = \exp\left[-8\pi E_2\left(M-E_1-\frac{E_2}{2}\right)\right]. \tag{3}$$

The tunneling probability for two emissions with energies E_1 and E_2, respectively, can be computed accordingly as

$$\Gamma(E_1,E_2)=\Gamma(E_1)\Gamma(E_2|E_1) \;=\; \exp\left[-8\pi(E_1+E_2)\left(M-\frac{E_1+E_2}{2}\right)\right]. \tag{4}$$

Interestingly, we find

$$\Gamma(E_1,E_2)=\Gamma(E_1+E_2), \tag{5}$$

or the tunneling probability of a particle with an energy E_1+E_2 is the same as the probability for two emissions of energies at E_1 and E_2.

To prove the existence of correlation between the two emissions and to properly quantify the amount of correlation, we need to find the independent probability for each emission. Using the theory of probability, we find the independent probability for the emission at energy E_1 to be $\Gamma_1(E_1)=\int\Gamma(E_1,E_2)dE_2$, which is identically the same as $\Gamma(E_1)$. For

the emission at energy E_2, we find $\Gamma_2(E_2) = \int \Gamma(E_1, E_2) dE_1$, which again takes the same functional form of $\Gamma(E_2)$. We find that

$$\ln \Gamma(E_1 + E_2) - \ln \left[\Gamma(E_1)\ \Gamma(E_2)\right] = 8\pi E_1 E_2 \neq 0, \tag{6}$$

which shows that the two tunnelings are not statistically independent, and there indeed exist correlations between Hawking radiations. As will become clear later, the existence of this correlation is central to the resolution we provide for the black hole information loss paradox.

A fundamental assumption in statistical mechanics concerns the equal probability distribution for every micro-state, which forms the basis of the micro-canonical ensemble approach. Given the quantum tunneling of an emitted particle with an energy E, or a Hawking radiation from a black hole, with the probability of Eq. (1), when the black hole is exhausted, we can find the entropy of the total system by counting the numbers of its microstates. For example, one of the microstates is $(E_1, E_2, \cdots, E_n)$ and $\sum_i E_i = M$. Within such a description, the order of E_i cannot be changed, the distribution of each E_i is consistent with the tunneling probabilities discussed in the main text. The probability for the specific microstate $(E_1, E_2, \cdots, E_n)$ to occur is given simply by

$$\begin{aligned} P_{(E_1,E_2,\cdots,E_n)} &= \Gamma(E_1, E_2, \cdots, E_n) \\ &= \Gamma(E_1) \times \Gamma(E_2|E_1) \times \cdots \times \Gamma(E_n|E_1, E_2, \cdots E_{n-1}). \end{aligned} \tag{7}$$

Following the steps involved in arriving at Eq. (5), it is easy to show

$$\Gamma(E_1, E_2, E_3) = \Gamma(E_1 + E_2, E_3) = \Gamma(E_1 + E_2 + E_3), \tag{8}$$

and analogous identities for all subsequent emissions. Finally we obtain

$$\begin{aligned} P_{(E_1,E_2,\cdots,E_n)} &= \Gamma(E_1, E_2, \cdots, E_n) \\ &= \Gamma(\sum_{j=1}^{N} E_j) = \Gamma(M) = \exp(-4\pi M^2) = \exp(-S_{\mathrm{BH}}). \end{aligned} \tag{9}$$

The total number of microstates is therefore given by

$$\Omega = \frac{1}{p_{(E_1,E_2,\cdots,E_n)}} = \exp(S_{\mathrm{BH}}). \tag{10}$$

According to the Boltzmann's definition, the entropy of a system is given by $S = \ln \Omega = S_{\mathrm{BH}}$, where the Boltzmann's constant is taken as unity for simplicity. Thus we show after a black hole is exhausted due to Hawking radiation, the entropy carried away in the emitted particles (Hawking radiations) is precisely equal to the entropy S_{BH} in the original black hole [1, 2, 27].

This result is in direct contradiction with the black hole information loss paradox. A lot of previous investigations [3, 28, 29, 30] support the claim that information can be lost in a black hole. Thus the total entropy of a black hole increases during Hawking radiation [31, 32]. The analysis we provide here, however, shows otherwise. Based on a straightforward calculation using statistical theory, we find the total entropy is conserved. This shows the time evolution of a black hole is unitary. In particular, the Hawking radiation remains governed by conservations laws we are accustomed to.

Two significant points distinguish our investigation from most existing theories. First, we start with the assumption of the non-thermal spectrum for Hawking radiation as derived by Parikh and Wilczek [14]. Second, we discover the existence of information-carrying correlations among different Hawking emissions assuming the nonthermal spectrum [25]. The physics of both points lie at the fundamental law of energy conservation. In contrast to the earlier thermal spectrum of Hawking, Parikh and Wilczek enforced energy conservation when they computed the emission spectrum based on Hawking radiation as tunneling. The arising of correlations between different emissions then becomes easy to understand as we have shown earlier: after the emission of a Hawking radiation, the mass of the remaining black hole decreases, which subsequently affects the next emission, thus establishes correlations between different emissions. Assuming the particular form of the non-thermal spectrum by Parikh and Wilczek, the correlations among different emissions of Hawking radiation can be thoroughly studied.

We next provide a careful quantification for the amount of the correlation between different Hawking emissions in terms of the amount of information it can encode. In addition to establishing an alternative proof, the discussion in the next few paragraphs provides an insightful understanding of our claim that Hawking radiation is a unitary process

and the entropy for a black hole plus its Hawking radiation is conserved. More vividly, our analysis below shows that Hawking radiations can be viewed as messengers. In this way, information is leaked out through correlated tunneling processes that will be shown clearly when we compare the amount of correlation with the mutual information between the two emissions.

Because of the existence of correlation, we shall be very careful when considering emissions of particles with energies E_1 and E_2, one after another, because $\Gamma(E_2|E_1)$ is the conditional probability for an emission at energy E_2 given the occurrence of an emission with energy E_1. More generally we use E_i to denote the energy for the ith emission. Given the total energy for all previous emissions $\sum E_i = E_1 + E_2 + \cdots + E_{f-1}$, the tunneling probability for a next emission of energy E_f becomes $\Gamma(E_f|E_1, E_2, \cdots, E_{f-1}) = \exp\left[-8\pi E_f\left(M - \sum E_i - \frac{E_f}{2}\right)\right]$. From $\Gamma(E_f|E_1, E_2, \cdots, E_{f-1})$, we can compute the entropy taken away by a tunneling particle with energy E_f after the black hole has emitted a total energy $\sum E_i$, which is given by

$$S\left(E_f\left|E_1, E_2, \cdots, E_{f-1}\right.\right) = -\ln\Gamma\left(E_f\left|E_1, E_2, \cdots, E_{f-1}\right.\right). \tag{11}$$

In quantum information theory, $S\left(E_f|E_1, E_2, \cdots, E_{f-1}\right)$ denotes conditional entropy and is used to quantify the remaining entropy of an emission with energy E_f given that information for all previously emitted particles with a total energy $\sum E_i$ are known. In quantitative terms, we find $S(E_f|E_1, E_2, \cdots, E_{f-1})$ is equal to the decrease of entropy for a black hole with a mass $M - \sum E_i$ upon an emission of E_f. This also agrees with the general second law of black hole thermodynamics [33, 34]. The tunneling particles must carry entropy with themselves because the total entropy of a black hole and its radiations can never decrease. In what follows we will show that by using entropy we can measure the amount of information hidden in the correlation (6).

The mutual information [35] between two subsystems A and B in a composite bi-partite system is defined as

$$S(A:B) \equiv S(A) + S(B) - S(A,B) = S(A) - S(A|B), \tag{12}$$

where $S(A|B)$ is nothing but the conditional entropy. This mutual information can be used to measure the total amount of correlations between any bi-partite systems. When applied to the emission of two particles with energies E_1 and E_2, their mutual information becomes

$$S(E_2:E_1) \equiv S(E_2) - S(E_2|E_1) = -\ln\Gamma(E_2) + \ln\Gamma(E_2|E_1). \tag{13}$$

For a classical horizon, using Eqs. (2) and (3), we find previously $S(E_2:E_1) = 8\pi E_1 E_2$. This shows that the correlation between emissions of Hawking radiation can carry information, *i.e.*, the above Eq. (13) affirms that the amount of correlation quantity in Eq. (6) is precisely equal to the mutual information between emissions for a classical horizon. Additionally, this justifies the reexamination of the entropy (11) by quantum information theory.

We now compute the entropy of all tunneled particles. The entropy for the first tunneled particle, with energy E_1 from a black hole of mass M, is given by

$$S(E_1) = -\ln\Gamma(E_1) = 8\pi E_1\left(M - \frac{E_1}{2}\right). \tag{14}$$

The entropy for the second tunneling particle with energy E_2 after the emission of a particle with energy E_1 becomes

$$S(E_2|E_1) = -\ln\Gamma(E_2|E_1) = 8\pi E_2\left(M - E_1 - \frac{E_2}{2}\right), \tag{15}$$

analogous to the formula (14), except for the reduced mass of the black hole to $M - E_1$ due to the first emission from energy conservation. The total entropy from the two emitted particles E_1 and E_2 is

$$S(E_1, E_2) = S(E_1) + S(E_2|E_1), \tag{16}$$

i.e., rightfully including the contribution from their correlation. This result can be repeated. After the tunneling of particles with energies E_1 and E_2, the mass of the black hole becomes $M - E_1 - E_2$, and it proceeds to emit a third particle with energy E_3 due to tunneling. The corresponding tunneling entropy is $S(E_3|E_1, E_2) = -\ln(E_3|E_1, E_2)$, which gives the total entropy for the three emissions, respectively, at energies E_1, E_2, and E_3,

$$S(E_1, E_2, E_3) = S(E_1) + S(E_2|E_1) + S(E_3|E_1, E_2). \tag{17}$$

For all emissions which eventually exhausts the black hole, we find

$$S(E_1, E_2, ..., E_n) = \sum_{i=1}^{n} S(E_i|E_1, E_2, \cdots, E_{i-1}), \tag{18}$$

where $M = \sum_{i=1}^{n} E_i$ is the initial energy of the black hole due to energy conservation. The generalized term $S(E_1, E_2, ..., E_n)$ denotes the joint entropy of all emitted radiations and $S(E_i|E_1, E_2, \cdots, E_{i-1})$ is the respective conditional entropy for the ith emission with energy E_i after the emissions of a total of $i-1$ particles. We recall that Eq. (18) satisfies the chain rule for conditional entropies in information theory (Please see Ref. [35], chapter 11). When the black hole is exhausted, all of its entropy is carried away by Hawking radiations. By a detailed calculation from Eq. (18), we previously show that the total entropy of all Hawking radiations equals to $S(E_1, E_2, ..., E_n) = 4\pi M^2$, which is exactly the same as the Bekenstein-Hawking entropy of a black hole. This equation shows that the entropy of a black hole is indeed taken out by Hawking radiations, and the total entropy of all emitted radiations and the black hole is unchanged during the black hole radiation process. According to quantum mechanics, only unitary processes conserve the entropy for a closed system. Thus we conclude that irrespective of the microscopic picture for a black hole, the fact that the total entropy remains conserved during Hawking radiation implies that the black hole radiation as tunneling is unitary in principle. This provides a self-consistent and reasonable resolution to the long standing paradox of black hole information loss.

Reissner-Nordström black hole

The method of null geodesic can also be applied to treat Hawking radiation from a charged black hole. The emission due to tunneling of non-charged (neutral) particles was considered in Ref. [14]. The counterpart to the Painlevé coordinate for the charged Reissner-Nordström coordinate is

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + 2\sqrt{\frac{2M}{r} - \frac{Q^2}{r^2}}\, dtdr + dr^2 + r^2 d\Omega^2. \tag{19}$$

Following the standard procedure, the imaginary part of the action for an outgoing massless neutral particle can be computed, and the resulting tunneling probability is

$$\begin{aligned} \Gamma &\sim \exp\left[-4\pi \left(2EM - E^2 - (M-E)\sqrt{(M-E)^2 - Q^2} + M\sqrt{M^2 - Q^2}\right)\right] \\ &= \exp(\Delta S), \end{aligned} \tag{20}$$

where $S = \pi(M + \sqrt{M^2 - Q^2}\,)^2$ is the entropy. The corresponding temperature for a charged black hole is $T = \frac{1}{2\pi} \frac{\sqrt{M^2-Q^2}}{(M+\sqrt{M^2-Q^2}\,)^2}$, that allows us to compare the tunneling probability (20) with the Boltzmann factor for emission $\Gamma = \exp(-\beta E)$, $(\beta = 1/T)$. As before, we find this spectrum is non-thermal, and the relationship $\Gamma(E_1 + E_2) = \Gamma(E_1, E_2)$ remains true. Using the same argument as before for computing the information or entropy carried by the correlations of two emitted particles for a Schwarzchild black hole outlined in the previous subsection, we find

$$\ln \Gamma(E_1 + E_2) - \ln \left[\Gamma(E_1)\ \Gamma(E_2)\right] \neq 0. \tag{21}$$

Again we find the existence of correlations among Hawking radiations because the spectrum for emission from a charged black hole is also non-thermal. In the process of two particles tunneling with respective energies E_1 and E_2, we find the entropy form

$$\begin{aligned} S(E_1) &= -\ln \Gamma(E_1) = 4\pi \left(2E_1 M - E_1^2 - (M-E_1)\sqrt{(M-E_1)^2 - Q^2} + M\sqrt{M^2 - Q^2}\right), \\ S(E_2|E_1) &= -\ln \Gamma(E_2|E_1) \\ &= 4\pi \left(2E_2 (M-E_1) - E_2^2 - (M-E_1-E_2)\sqrt{(M-E_1-E_2)^2 - Q^2} + M\sqrt{M^2 - Q^2}\right). \end{aligned}$$

Not surprisingly, they satisfy the definition of conditional entropy, $S(E_1, E_2) = -\ln \Gamma(E_1 + E_2) = S(E_1) + S(E_2|E_1)$. After a detailed calculation, again we find that the amount of correlation is exactly equal to the mutual information described in Eq. (13), and this shows that the correlation can carry information from a Reissner-Nordström black

hole and any single step of the emission must be entropy preserving. We can count the total entropy carried away by all outgoing particles, and find

$$S(E_1, E_2, \cdots, E_n) = \sum_{i=1}^{n} S(E_i|E_1, E_2, \cdots, E_{i-1}) = S(E), \tag{22}$$

where $E = \sum_{i=1}^{n} E_i$ is the total energy of the black hole radiation.

A subtle difference arises from the previously considered Schwarzchild black hole. In the present case, the mass of a black hole can never decrease to zero, yet the tunneling probability must remain a real value, thus $M^2 - Q^2 \geqslant 0$ is to be enforced. Because the tunneled particles are taken as neutrals, the extreme case is reached when $M^2 = Q^2$. The temperature is $T = 0$ in the extreme case, so Hawking radiation will vanish and the black hole is stablized. This is consistent with cosmic censorship. For the information loss paradox we seek to resolve, it is important to ask: is entropy still conserved in the extreme case? The key point to a legitimate answer depends on how to properly describe the entropy of an extreme black hole. According to the definition of Bekenstein-Hawking, the entropy of an extreme black hole is proportional to its surface area, although its temperature is zero. Thus we can conclude that even in the extreme case the entropy remains conserved. Although the entropy (or the information) cannot be taken out completely, the residual information remains inside the extreme black hole. This is reasonable because the extreme limit can be viewed as the ground state of a charged black hole, which has a high degeneracy $\sim e^{S_e}$ with $S_e = \pi M^2$ [36].

A microscopic picture of tunneling by charged particles is more appropriate in order to avoid the extreme case. Fortunately, the tunneling probability of charged massive particles for a Reissner-Nordström black hole has been obtained in Ref. [17]. With outgoing particles capable of carrying away charges from a charged black hole, its semi-classical trajectories have to be modified due to electromagnetic forces. In the treatment of Ref. [17], the 4-dimensional electromagnetic potential $A_\mu = (A_t, 0, 0, 0)$ where $A_t = -Q/r$ is introduced in the quasi-Painlevé coordinates (19) and the electromagnetic interaction $-(1/4)F_{\mu\nu}F^{\mu\nu}$, which can be described by the potential A_μ, also has to be considered in calculating the action. Taking into account the modifications to the equation of motion due to the change of charge and including the contribution from the electromagnetic interaction, the tunneling probability is found to be [17],

$$\begin{aligned} \Gamma &\sim \exp\left[\pi\left(M - E + \sqrt{(M-E)^2 - (Q-q)^2}\right)^2 - \pi\left(M + \sqrt{M^2 - Q^2}\right)^2\right] \\ &= \exp\left(\Delta S\right), \end{aligned} \tag{23}$$

where $\Delta S = S(M - E, Q - q) - S(M, Q)$ is the difference of entropies for a Reissner-Nordström black hole before and after the emission, and q is the charge that is carried away by the particle with energy E. Comparing with the Boltzmann factor, we find clearly this remains a non-thermal spectrum. Using our method outlined before, we conclude there exists information-carrying correlations among emitted particles. Analogously, after a detailed calculation, we find that the total entropy carried away by the outgoing particles plus that of the accompanying black hole remains conserved, which is now due to both energy and charge conservations. Thus, we find once again no information is lost in the Hawking radiation as tunneling for a Reissner-Nordström black hole.

Kerr black hole

For rotating black holes [18], a complication arises from the frame-dragging effect of the coordinate system in the stationary rotating spacetime. The matter field in the ergosphere near the horizon must be dragged by the gravitational field with an azimuthal angular velocity. A proper physical picture thus must be capable of describing such effects in the dragged coordinate system. Adopting the quasi-Painlevé time transformation and the dragging coordinate transformation for the Doran form of the Kerr coordinates [37], the so-called dragged Painlevé-Kerr coordinates can be expressed as

$$ds^2 = \frac{\triangle\Sigma}{(r^2+a^2)^2 - \triangle a^2\sin^2\theta}\,dt^2 - \frac{\Sigma}{r^2+a^2}dr^2 - 2\frac{\sqrt{2Mr(r^2+a^2)}\Sigma}{(r^2+a^2)^2 - \triangle a^2\sin^2\theta}\,dtdr - \Sigma\,d\theta^2, \tag{24}$$

where a is the angular momentum of a unit mass, $\Sigma = r^2 + a^2\cos^2\theta$, and $\triangle = r^2 + a^2 - 2Mr$. In these coordinates, the event horizon and the infinite red-shifted surface coincides with each other so that the WKB approximation can

be used to calculate the imaginary part of the action. The tunneling probability for a rotating black hole is then found to be [18]

$$\begin{aligned}\Gamma &\sim \exp\left[-2\pi\left(M^2-(M-E)^2+M\sqrt{M^2-a^2}-(M-E)\sqrt{(M-E)^2-a^2}\right)\right]\\ &= \exp(\Delta S)\,,\end{aligned} \tag{25}$$

where $\Delta S = S(M-E, Q-q) - S(M,Q)$ is the difference of the entropies for a Kerr black hole before and after the emission of a particle with energy E. This spectrum (25) is once again non-thermal, and the interesting property $\Gamma(E_1+E_2) = \Gamma(E_1, E_2)$ remains true. The total amount of correlations hidden inside Hawking radiations can again be computed analogously, and we find

$$\ln\Gamma(E_1+E_2) - \ln\left[\Gamma(E_1)\ \Gamma(E_2)\right] \neq 0. \tag{26}$$

For tunneling of two particles with respective energies E_1 and E_2, we find the entropies

$$S(E_1) = -\ln\Gamma(E_1) = 2\pi\left(M^2-(M-E_1)^2+M\sqrt{M^2-a^2}-(M-E_1)\sqrt{(M-E_1)^2-a^2}\right), \tag{27}$$

$$\begin{aligned}S(E_2|E_1) = -\ln\Gamma(E_2|E_1) &= 2\pi\left[(M-E_1)^2-(M-E_1-E_2)^2+(M-E_1)\sqrt{(M-E_1)^2-a^2}\right]\\ &\quad -2\pi\left[(M-E_1-E_2)\sqrt{(M-E_1-E_2)^2-a^2}\right].\end{aligned} \tag{28}$$

Clearly they also satisfy the definition of conditional entropy $S(E_1,E_2) = -\ln\Gamma(E_1+E_2) = S(E_1)+S(E_2|E_1)$. A detailed calculation again reveals that the amount of correlation in this case is exactly equal to the mutual information described in Eq. (13). We can count the total entropy carried away by the outgoing particles in the same manner and find

$$S(E_1, E_2, ..., E_n) = \sum_{i=1}^{n} S(E_i|E_1, E_2, \cdots, E_{i-1}) = S(E), \tag{29}$$

where $E = \sum_{i=1}^{n} E_i$ is the total energy of the Hawking radiations.

After a detailed calculation, we again find that no information is lost as the tunneling process is an entropy conserving one. However, since the black hole is rotating, angular momentum conservation must be considered. In the tunneling process considered here, we do not see how the angular momentum is carried away. An obvious reason is that the total angular momentum of a black hole is absent in the coordinates and instead the angular momentum of a unit mass is used. The outgoing particles clearly carry away the angular momentum and this can be seen in the calculation for the imaginary part of the action and the tunneling probability. We conclude that the entropy conservation is due to both energy conservation and angular momentum conservation. Once again, it is the existence of information-carrying correlations due to energy conservation that resolves the information loss paradox for Hawking radiation as tunneling for a Kerr black hole.

We note that due to the angular momentum conservation of a unit mass in the tunneling process, the extreme case $a^2 = M^2$ is guaranteed to appear and the radiation will then stop. However this does not contradict our conclusion because in the extreme limit all tunneling processes vanish and the residual entropy will remain inside the extreme black hole, which is consistent with entropy conservation and unitarity.

Kerr-Newman black hole

If we were to consider uncharged massless particles tunneling from a charged rotating black hole, the calculation would completely parallel that for a Kerr black hole. Instead, we consider tunneling of charged massive particles for a Kerr-Newman black hole [18, 19]. Like for a Kerr black hole, one has to treat the frame-dragging effect using dragging coordinate transformation, and then the event horizon becomes consistent with the infinite red-shifted surface, which allows for the WKB approximation to be used. On the other hand, because the particles are now charged, the contribution to the action from the electromagnetic interaction has to be included.

Performing the quasi-Painlevé time transformation and the dragging coordinate transformation for the Boyer-Lindquist form of the Kerr-Newman coordinates [38], the dragged Painlevé-Kerr-Newman coordinates are obtained as following

$$ds^2 = \frac{\triangle\Sigma}{\left(r^2+a^2\right)^2 - \triangle a^2 \sin^2\theta}\, dt^2 - \frac{\Sigma}{r^2+a^2}\, dr^2 - 2\frac{\sqrt{\left(2Mr-Q^2\right)\left(r^2+a^2\right)}\,\Sigma}{\left(r^2+a^2\right)^2 - \triangle a^2 \sin^2\theta}\, dtdr - \Sigma\, d\theta^2. \tag{30}$$

Calculating the imaginary part of the action in the usual manner including the contribution from electromagnetic interaction, the tunneling probability of charged massive particles for a Kerr-Newman black hole [18, 19] is found to be

$$\begin{aligned}\Gamma \;&\sim\; \exp\left[\pi\left(M-E+\sqrt{(M-E)^2-(Q-q)^2-a^2}\right)^2 - \pi\left(M+\sqrt{M^2-Q^2-a^2}\right)^2\right] \\ &=\; \exp\left(\Delta S\right),\end{aligned} \tag{31}$$

where $\Delta S = S(M-E, Q-q) - S(M,Q)$ is the difference of entropies for a Kerr-Newman black hole before and after the emission of a particle with energy E and charge q. Not surprisingly, the spectrum is again a non-thermal one, and an analogous relationship $\Gamma(E_1+E_2, q_1+q_2) = \Gamma(E_1, q_1, E_2, q_2)$, from which we can affirm the existence of correlation between two radiated emissions as

$$\ln\Gamma(E_1+E_2, q_1+q_2) - \ln\left[\Gamma(E_1,q_1)\ \Gamma(E_2,q_2)\right] \neq 0. \tag{32}$$

Like other types of black holes we considered previously, we find that there exists correlation in Hawking radiation from a charged rotating black hole. For two emissions with respective energies, E_1 and E_2, and charges, q_1 and q_2, we find

$$\begin{aligned}S(E_1,q_1) = -\ln\Gamma(E_1,q_1) \;&=\; \pi\left(M-E_1+\sqrt{(M-E_1)^2-(Q-q_1)^2-a^2}\right)^2 \\ &\quad -\pi\left(M+\sqrt{M^2-Q^2-a^2}\right)^2,\end{aligned} \tag{33}$$

$$\begin{aligned}S(E_2,q_2|E_1,q_1) = -\ln\Gamma(E_2,q_2|E_1,q_1) \;&=\; \pi\left(M-E_1-E_2+\sqrt{(M-E_1-E_2)^2-(Q-q_1-q_2)^2-a^2}\right)^2 \\ &\quad -\pi\left(M-E_1+\sqrt{(M-E_1)^2-Q^2-a^2}\right)^2.\end{aligned} \tag{34}$$

They satisfy the definition for conditional entropy $S(E_1,q_1,E_2,q_2) = -\ln\Gamma(E_1+E_2,q_1+q_2) = S(E_1,q_1) + S(E_2,q_2|E_1,q_1)$. After a detailed calculation, we again find the amount of correlation is exactly equal to the mutual information described in Eq. (13). We can count the total entropy carried away by the outgoing particles in the same manner, and we find

$$S(E_1,q_1,E_2,q_2,...,E_n,q_n) = \sum_{i=1}^{n} S(E_i,q_i|E_1,q_1,E_2,q_2,\cdots,E_{i-1},q_{i-1}) = S(E,q), \tag{35}$$

where $E=\sum_{i=1}^n E_i$ and $q=\sum_{i=1}^n q_i$ is the total energy and charge of the Hawking radiations. This shows entropy conservation in the Hawking radiation for a Kerr-Newman black hole. Clearly the conservation of entropy arises because of energy conservation, charge conservation, and angular momentum conservation being rightfully enforced for the process. We thus find no information is lost in Hawking radiation as tunneling for a Kerr-Newman black hole. Like the situation for the Kerr black hole, the angular momentum of a unit mass remains a constant, or conserved in the tunneling process, the extreme case $a^2 = M^2$ thus must appear in the end of tunneling process. However, this doesn't contradict our conclusions because the Hawking radiation terminates in the extreme case, and the residual entropy will remain a constant inside the black hole.

Before concluding this section, we will explain why information can be carried away from a black hole by Hawking radiation as tunneling. The most important reason is that the emission process is probabilistic, not a deterministic one. For each tunneling emission from a black hole, we only know a radiation may occur with a probability $\Gamma(E)$, nothing else. In other words, the uncertainty of the event (for a radiation with energy E) or the potential information we can gain from the event is $S(E) = -\ln\Gamma(E)$.

When a radiation with energy E_1 is received, the potential information we can gain is $S(E_1) = -\ln\Gamma(E_1)$. After already receiving the emission at energy E_1, when we receive the next radiation with energy E_2, the potential information we can gain is $S(E_2|E_1) = -\ln\Gamma(E_2|E_1)$. Step by step, we can track each subsequent emission, all of the n emissions until the black hole stops radiating. Of course, we have to assume that the observer is rightfully equipped to detect all radiations. In the end, the information gained by the observer is $S(E_1, E_2, ..., E_n)$, where for a Schwarzchild black hole, $S(E_1, E_2, ..., E_n)$ is nothing but the initial entropy of the black hole. For a Reissner-Nordström black hole, a Kerr black hole, and a Kerr-Newman black hole, the entropy of their extreme black holes is $S_e = S_{\rm BH} - S(E_1, E_2, ..., E_n)$, where $S_{\rm BH}$ is their respective initial entropy. However, given all radiations are received and detected by an observer, he/she cannot reconstruct the initial state from which the matter collapses into a black hole. A legitimate reconstruction may need more knowledge concerning the dynamic description for black hole radiation, which seems only possible with a complete theory for quantum gravity.

In concluding this section, we use statistical method and quantum information theory to show that no information is lost in Hawking radiation as tunneling. This result constrains the black hole evaporation as tunneling to be a unitary process. Our conclusion is based on a single but an important observation, that a non-thermal Hawking radiation spectrum implies the existence of information-carrying correlation among emitted particles. Upon counting the total entropy, the portion due to correlation is found to exactly balance the part previously perceived as lost. Within our suggested resolution, we find that energy conservation or the self-gravitation effect plays a crucial role. For some black holes such as charged, rotating, and charged-rotating black holes, other conservation laws, such as charge conservation and angular momentum conservation, also affect the tunneling rate and are important for the entropy conservation. This implies that in further studies about the black hole information loss paradox, quantum gravity theory should be considered within the framework of energy, charge, and angular momentum conservations.

HAWKING RADIATION AS TUNNELING THROUGH A QUANTUM HORIZON

Hawking radiation as tunneling through a quantum horizon has been considered before [20], and the tunneling probability is already given in a general spherically symmetric system in the ADM form [16] by referencing to the first law of black hole thermodynamics $dM = \frac{\kappa}{2\pi} dS$,

$$\Gamma \sim (1 - \frac{E}{M})^{2\alpha} \exp\left[-8\pi E\left(M - \frac{E}{2}\right)\right] = \exp\left(\Delta S\right), \tag{36}$$

where $S = \frac{A}{4} + \alpha \ln A$ is the entropy derived by directly counting the number of micro-states with string theory and loop quantum gravity [20]. The coefficient α is negative in loop quantum gravity [39]. Its sign remains uncertain in string theory, depending on the number of field species in the low energy approximation [40]. When $\alpha > 0$ and the tunneling energy approaches the mass of the black hole, the tunneling probability $\Gamma \to 0$. A more interesting case occurs when $\alpha < 0$, the black hole will not radiate away all of its mass. The tunneling will halt at a critical value of the black hole mass giving rise to a situation similar to a black hole remnant as described in Ref. [41]. Recently, some quantum properties of the Bekenstein-Hawking entropy and its universal sub-leading corrections have been discussed [42, 43, 44].

For Hawking radiation as tunneling through a quantum horizon as described by the Eq. (36), we can use the same statistical method as described by Eqs. (2) and (4) to probe and measure correlation. We find as before the interesting relationship $\Gamma(E_1 + E_2) = \Gamma(E_1, E_2)$ remains true. The amount of correlation is evaluated to be

$$\ln\Gamma(E_1 + E_2) - \ln\left[\Gamma(E_1)\ \Gamma(E_2)\right] = 8\pi E_1 E_2 + 2\alpha \ln \frac{M(M - E_1 - E_2)}{(M - E_1)(M - E_2)} \neq 0. \tag{37}$$

Once again, correlations are found to exist due to the non-thermal nature of the spectrum Eq. (36), despite being corrected by quantum gravity effect. Unlike the result of Eq. (6), an additional term appears as the second term before the last inequality sign in Eq. (37). On careful examination, we conjecture that this correction may carry information about effects of quantum gravity or black hole area quantization [42, 43, 44].

The process of two emissions can be considered as in the situation when the Bekenstein-Hawking entropy is used, and we find

$$S(E_1) = -\ln\Gamma(E_1) \ = \ 8\pi E_1 (M - \frac{E_1}{2}) - 2\alpha \ln(1 - \frac{E_1}{M}), \tag{38}$$

$$S(E_2|E_1) = -\ln\Gamma(E_2|E_1) \ = \ 8\pi E_2 (M - E_1 - \frac{E_2}{2}) - 2\alpha \ln\left(1 - \frac{E_2}{M - E_1}\right). \tag{39}$$

Again this form is consistent with the definition of conditional entropy $S(E_1, E_2) = S(E_1) + S(E_2|E_1)$. Repeating the steps until the black hole is exhausted by emissions, we find

$$S(E) = \sum_{i=1}^{n} S(E_i|E_1, E_2, \cdots, E_{i-1}), \tag{40}$$

where $E = \sum_{i=1}^{n} E_i$ is the total energy of the black hole radiation.

For $\alpha > 0$, we find $\Gamma(E) \to 0$ when $E \to M$, but $S(M - E) \to \infty$. This causes difficulty explaining the origin of an exponentially growing entropy when the black hole vanishes. However, qualitatively, this actually can be understood within the picture of Hawking radiation from a black hole. In the limit of $\Gamma(M) = 0$, the tunneling energy approaches the mass of the black hole, and the tunneling becomes slower and slower while the time to exhaust a black-hole approaches infinite. This infinity also can be obtained from other methods by using the Stefan-Boltzmann law as in Ref. [45].

For $\alpha < 0$, it is known [41] that when the mass of a black hole approaches the critical mass M_c, no particles will be emitted. From Eq. (40) we then obtain $S(M) - S(M_c) = \sum\limits_i S(E_i|E_1, E_2, \cdots, E_{i-1})$ or $S(M) = \sum\limits_i S(E_i|E_1, E_2, \cdots, E_{i-1}) + S(M_c)$. In Ref. [41], the mass M_c is called the "zero point energy" of a black hole that is similar to a black hole remnant because it does not depend on the initial black hole mass. We have shown that even with such a remnant, the total entropy remains conserved when information carried away by correlations are correctly included. Thus the unitarity remains true when the classical horizon is replaced by a quantum one for Hawking radiation.

HAWKING RADIATION AS TUNNELING FOR A NON-COMMUTATIVE BLACK HOLE

This section is devoted to the consideration of information loss paradox for a black hole in non-commutative Schwarzschild spacetime [21]. From the result of the third section above, the effective temperature $T = \frac{1}{8\pi M}\left(1 + \frac{\alpha}{M^2}\right)$ will diverge when a black hole is exhausted due to Hawking radiation when the effect of quantum gravity or reaction is included. If the coefficient α happens to take a negative value, the trouble with a diverging temperature is avoided because the black hole will not evaporate to exhaustion. Instead, the radiation ends with a remnant left behind [41]. Various versions of string theories, however, predict a positive value for the coefficient α, thus leaving the divergent temperature an unresolved issue. A plausible solution is to adopt a non-commutative spacetime in order to remove the so called Hawking paradox when the temperature for a standard black hole diverges as its radius shrinks to zero. In a non-commutative space, the mass density of point particles cannot be expressed simply as a product of its mass with the Dirac delta function for the space coordinate as is often used in commutative spaces. The non-commutativity introduces a fuzziness of space, something like a position-position uncertainty relationship. This causes several corrections to the expression for the mass density. One of the greatest changes concerns the mass of a black hole, it is no longer located at a point, instead it is smeared around a region of space [46, 47]. This correction to the mass of a black hole is proportional to the non-commutative parameter, an analogous parameter to the role $\hbar$ plays in quantum theory. The resulting correction vanishes, *i.e.*, everything reduces to their normal forms in a commutative spacetime, when this parameter approaches zero. After some rearrangements, we find that the non-commutative Schwarzschild metric takes the form

$$ds^2 = -\left[1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)\right] dt^2 + \left[1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)\right]^{-1} dr^2 + r^2 d\Omega^2, \tag{41}$$

where θ is the non-commutative parameter, and $\gamma(a, x) = \int_0^x t^{a-1}e^{-t}dt$ is the lower incomplete gamma function. Transforming this metric into the quasi-Painlevé metric and including the reaction effect, the imaginary part of the action can be calculated to arrive at the tunneling probability as [21]

$$\begin{aligned} \Gamma \ \sim \ & \left[1 - \frac{2E\left(M - \frac{E}{2}\right)}{M^2 + \alpha}\right]^{-4\pi\alpha} \exp\left[-8\pi E\left(M - \frac{E}{2}\right)\right] \times \\ & \exp\left[16\sqrt{\frac{\pi}{\theta}}M^3 e^{-\frac{M^2}{\theta}} - 16\sqrt{\frac{\pi}{\theta}}\left(M - E\right)^3 e^{-\frac{(M-E)^2}{\theta}} + \text{const.}\,(\text{independent of } M)\right]. \end{aligned} \tag{42}$$

The first exponential factor of Eq. (42) was previously obtained by Parikh and Wilczek where neither non-commutativity nor back reaction effects were considered. The remaining parts, the prefactor and the second exponential, are actually due to the effect of back reaction and non-commutativity [21], which change the form of the entropy to

$$S \backsimeq 4\pi M^2 - 4\pi\alpha \ln\left(\frac{M^2}{\alpha}+1\right) - 16\sqrt{\frac{\pi}{\theta}} M^3 e^{-\frac{M^2}{\theta}} + \text{const.}\,(\text{independent of } M).$$

It is easily seen that the spectrum of Eq. (42) is non-thermal. Thus correlations are again important and exist between emitted radiations. Further, we find the interesting relationship $\Gamma(E_1+E_2)=\Gamma(E_1,E_2)$ again holds. For two emissions with energies E_1 and E_2, we find that

$$\ln\Gamma(E_1+E_2) - \ln\left[(\Gamma(E_1)\ \Gamma(E_2)\right] \neq 0. \tag{43}$$

Thus the adoption of a non-commutative spacetime does not change our statement that a non-thermal spectrum affirms the existence of correlation, as is illustrated here for a Schwarzschild black hole when the reaction effect is included. We find that the information associated with non-commutativity is factored out in the correlation even in the early stage of Hawking radiation. Thus even though non-commutativity only exists at the small scale, we can still test its effect through correlations contained in the non-thermal spectrum of Hawking radiation. For tunneling of two particles with energies E_1 and E_2, we find the entropy

$$\begin{aligned} S(E_1) = -\ln\Gamma(E_1) &= 8\pi E_1\left(M-\frac{E_1}{2}\right) + 4\pi\alpha\ln\left[1-\frac{2E(M-\frac{E_1}{2})}{M^2+\alpha}\right] \\ &\quad -16\sqrt{\frac{\pi}{\theta}}M^3 e^{-\frac{M^2}{\theta}} + 16\sqrt{\frac{\pi}{\theta}}\left(M-E\right)^3 e^{-\frac{(M-E)^2}{\theta}} - \text{const.}, \qquad (44) \\ S(E_2|E_1) = -\ln\Gamma(E_2|E_1) &= 8\pi E_2\left(M-E_1-\frac{E_2}{2}\right) + 4\pi\alpha\ln\left[1-\frac{2E_2(M-E_1-\frac{E_2}{2})}{\left(M-E_1\right)^2+\alpha}\right] \\ &\quad -16\sqrt{\frac{\pi}{\theta}}\left(M-E_1\right)^3 e^{-\frac{(M-E_1)^2}{\theta}} + 16\sqrt{\frac{\pi}{\theta}}\left(M-E_1-E_2\right)^3 e^{-\frac{(M-E_1-E_2)^2}{\theta}} - \text{const.} \qquad (45) \end{aligned}$$

As before, they satisfy the definition for conditional entropy $S(E_1,E_2) = -\ln\Gamma(E_1+E_2) = S(E_1)+S(E_2|E_1)$. A detailed calculation confirms that the amount of correlation is exactly equal to the mutual information described in Eq. (13), and this shows that it is the correlation that carries away the information. If we count the total entropy carried away by the outgoing particles, we find

$$S(E_1,E_2,...,E_n) = \sum_{i=1}^{n} S(E_i|E_1,E_2,\cdots,E_{i-1}). \tag{46}$$

Thus we show that entropy is conserved in Hawking radiation for a Schwarzschild black hole in a non-commutative spacetime. Our resolution of no information loss for this case, however, does not solve the remaining problem of whether the black hole will evaporate to exhaustion or will halt at some value of a critical mass because of the reaction effect or the use of a non-commutative spacetime. As we discussed before, when the quantum reaction parameter α is negative, a black hole will leave behind a remnant instead of radiating into exhaustion [41]. When non-commutative spacetime is introduced, however, the parameter α cannot be negative, in order to avoid a divergent temperature at the end of radiation. When the non-commutative parameter $\theta \neq 0$, a reasonable value for α would be positive, and thus when the mass of the black hole is reduced to a certain value, the temperature will drastically decrease to zero to form an extreme black hole [21]. Despite these subtleties, we have shown conclusively in this section that the black hole information loss paradox is resolved for the non-commutative Schwarzschild black hole by taking into account information carried away by correlations in emitted particles.

CONCLUSIONS

In this work, we have significantly expanded our self-consistent theory for the resolution of the paradox for black hole information loss. In the picture of Hawking radiation as tunneling [14], we have earlier pointed out the existence

of correlations among radiations whenever the emission spectrum or the tunneling probabilities are non-thermal. While phenomenological, this resolution, first proposed by us in an earlier communication [25], is firmly supported by statistical theory. Although we reply on the results from a semi-classical treatment of Hawking radiation as tunneling, there is no room for comprise regarding our main conclusion for the existence of correlation as we have shown here for the various type of black holes. Whenever the Hawking radiation spectrum takes a non-thermal form, correlations must exist among radiated particles, irrespective of the nature for these emissions being charged or neutral, massive or massless particles, etc.

By comparing the amount of information that can be encoded into this correlation with the mutual information of quantum information theory as we did previously [25], we have shown in this work for an extensive list of black holes that the amount of information is always precisely equal to the mutual information. Due to the tunneling or the emission, the mass of a black hole decreases, that lowers the entropy of a black hole. According to the general second law of thermodynamics, the total entropy of the system, consisting of a black hole and its radiation, can never decrease. Thus, the tunneled particles as Hawking radiation must carry away entropy. Upon careful evaluation of the total entropy for radiated particles, including the contribution of the correlation, we find that the total entropy is conserved in the tunneling process, which supports the statement of unitary evolution for Hawking radiation as tunneling [25]. In addition to the standard Schwarzschild black holes we considered earlier [25], the list of black holes we consider in this article includes Reissner-Nordström black holes, Kerr black holes, and Kerr-Newman black holes. Surprisingly, or perhaps not so surprisingly, even when considering the corrections due to quantum gravity [48] in the tunneling process, our conclusions remain true: there exists correlation among Hawking radiations if the emission spectrum is non-thermal; the total amount of correlation exactly balances the mutual entropy; the Hawking radiation is an entropy conserving process; and no information loss is expected. Within the framework of our result, we shall view the tunneling particles and the correlations as messengers capable of carrying away entropy and information to assure that the entropy for the total system is conserved or no information loss occurs. We have further extended our investigation into a non-commutative spacetime for a Schwarzschild black hole, where we again find that most of our conclusions remain true: the modified Hawking radiation spectrum is non-thermal; correlation exists and the amount of the correlation is equal to the mutual information; while the actual evaporation due to Hawking radiation becomes more complicated, in this case, we can again avoid the paradox of information loss through a recovery of the correlations among Hawking radiations.

Based on our current understanding, we feel our conclusions are significant at least in two aspects. The first concerns black hole thermodynamics. It has been shown before that the tunneling process we discuss satisfies the first law of black hole thermodynamics irrespective of whether the horizon is classical [49] or quantum [50]. No conclusive consensus exist concerning the second law of black hole thermodynamics. Before our work, it was not known how entropy changes, and our conclusion that Hawking radiation is an entropy conserving process is thus quite exciting. This opens the door to reversibility and unitary dynamics for a black hole in principle. We note that reversibility is consistent with microscopic unitarity. The second aspect concerns the information loss paradox. There are two important results here: (1) we have shown within the picture of Hawking radiation as tunneling that the entropy growth can be exactly balanced, which provides a necessary condition to resolve the Hawking paradox of black hole information loss. Any legitimate attempt to resolve this paradox must find a solution to balance the entropy growth during the black hole evaporation process if individual emissions are considered independent. If entropy is indeed growing in a process, this process cannot be unitary. (2) We have shown that there exists correlation among Hawking radiations from a black hole. Using quantum information theory, the mutual information between two quantum subsystems or two emitted particles as considered in this study, we show that the amount of correlation among the black hole radiations can encode exactly the same amount of information and carry them away upon emission, which certainly represents a plausible resolution to release the information locked in a black hole. Any resolution to the information loss paradox of a black hole requires such a suitable mechanism to release the information locked in a black hole. Fortunately, we have uncovered such a mechanism inherent in the correlations of emitted particles.

In summary, we have shown that entropy is conserved in Hawking radiation. This resolves the paradox of black hole information loss. The amount of information that formerly was perceived to be lost is found to be encoded and carried away by Hawking radiations. This leads us to affirm that Hawking radiation as tunneling is unitary, in principle, based on the work we presented in this article.

As a final remark we note that for some cases we study, a black hole does not radiate away all of its mass and leaves behind a remnant in the end or forms an extreme black hole. Even for these special cases, our study shows that our conclusions remain valid. Information is still leaked out so long as a black hole starts to radiate. Based on our theoretical treatment, or the resolution we present for the paradox of black hole information loss due to Hawking radiation, we find that energy conservation or self-gravitation effect should be enforced when quantum theory is unified with gravity. Conservation laws are the most fundamental elements in a consistent theory for quantum gravity.

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